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Natural convection in a horizontal cylindrical annulus using porous fins

S. Kiwan and O. Zeitoun
Mechanical Engineering Department, King Saud University, Riyadh, Saudi Arabia

Abstract

Purpose – The aim is to study the effects of fin conductivity ratio, Darcy number, and Rayleigh number on the average Nusselt number for fins made of porous material when attached to the inner cylinder of the annulus between two concentric cylinders. The paper also aims to compare the results with those obtained using solid fins over a range of Rayleigh numbers.

Design/methodology/approach – The Darcy-Brinkman equations were used to model the fluid flow inside the porous media and the Boussinesq approximation was used to model the buoyancy effect. The energy equation is also solved to find the temperature distribution in the domain of interest. The model equations are solved numerically using a finite volume code.

Findings – Porous fins provided higher heat transfer rates than solid fins for similar configurations. This enhancement in heat transfer reached 75 per cent at $Ra = 5 \times 10^4$ and $Da = 2.5 \times 10^{-2}$. It is also found that unlike solid fins the rate of heat transfer from the cylinder equipped with porous fins decreases with increasing the fin inclination angle.

Research limitations/implications – The range of the Rayleigh number considered in this research covers only the laminar regime. The research does not cover turbulent flows. In addition to that, the local thermal equilibrium assumption is used.

Practical implications – This work can help designers in selecting the proper material properties and operating conditions in designing porous fins to enhance the heat transfer in the annulus between two horizontal concentric cylinders under natural convection condition.

Originality/value – This work has not been done before and it can initiate additional research projects as looking at the performance of porous fins under other conditions and configurations (e.g. turbulent conditions).

Keywords Convection, Finite volume methods, Numerical analysis

Paper type Research paper

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Nomenclature

C_F = Forchheimer coefficient
c_p = specific heat (J kg^{-1} K^{-1})
D_i = inner annulus diameter (m)
D_o = outer annulus diameter (m)
d = fin thickness (m)
Da = Darcy number (K/D_i^2)
$\bar{h}$ = average heat transfer coefficient (W m^{-2} K^{-1})
h = local heat transfer coefficient (W m^{-2} K^{-1})
K = permeability (m^2)
k = thermal conductivity (W m^{-1} K^{-1})
k_{eff} = effective thermal conductivity (W m^{-1} K^{-1})
k_r = thermal conductivity ratio (k_{eff}/k_f)
L_f = length of the fin (m)
L_R = fin length ratio, $2L_f/(D_o - D_i)$
$\overline{Nu}$ = average Nusselt number $(\bar{h}D_i/k_f)$
q'' = heat flux (W/m^2)
Pr = Prandtl number (ν_f/α_f)

Ra = Rayleigh number $(g\beta\Delta T s^3/\alpha_f \nu_f)$
T = temperature (K)
T_i = temperature of annulus inner surface (K)
T_o = temperature of annulus outer surface (K)
T^* = non-dimensional temperature $((T - T_o)/(T_i - T_o))$
u = velocity in x-direction (m/s)
U = non-dimensional velocity in x-direction (uD_i/α_f)
v = velocity in y-direction (m/s)
V = non-dimensional velocity in y-direction (vD_i/α_f)
x = radial coordinate along fin (m)
X = non-dimensional axial coordinate (x/D_i)
y = coordinate normal to fin (m)
Y = non-dimensional lateral coordinate (y/D_i)

Greek symbols
α = thermal diffusivity, m^2/s
β = thermal expansion coefficient, 1/K
ε = porosity
Λ = Forchheimer number, $(C_F/\sqrt{Da})$
μ = dynamic viscosity, kg/m s
ν = kinematic viscosity, (m^2/s)
φ = angle as shown in Figure 1
θ = fin inclination angle, deg.
ρ = density, m^3/kg

Subscripts
1 = fluid domain
2 = porous domain
eff = effective
f = fluid
i = inner
o = outer
s = solid

Introduction

A considerable work was done on investigating natural convection in horizontal annuli without fins (Grigull and Hauf, 1966; Kuehn and Goldstein, 1974, 1976; Alshahrani and Zeitoun, 2005). Previous investigations in bare annuli showed limited heat transfer in the annulus. Thorough literature survey showed that comparatively little work was focused on natural convection in annuli with fins on the inner cylinders (Chai and Patankar, 1993; Farinas *et al.*, 1997, 1999; Rahnama *et al.*, 1999, 2002; Alshahrani and Zeitoun, 2006). The presence of internal fins alters the flow patterns and temperature distributions that consequently affects heat transfer coefficient in the annuli. The enhancement of heat transfer from fins is an important factor that has captured the interest of many researchers. This enhancement can be accomplished through the following techniques:

- increasing the surface area to volume ratio;
- increasing the thermal conductivity of the fin; and
- increasing the convective heat transfer coefficient between the surface of the fin and the surrounding fluid.

Changing heat transfer rate in fixed-flow geometry of two concentric cylinders can be done through the application of some radial fins to the surface of the hot cylinder. Chai and Patankar (1993) were one of the first who investigated the effect of radial fins on laminar natural convection in horizontal annuli. They considered an annulus with six radial fins attached to the inner cylinder. They studied the effects of two fin orientations; the first is when two fins of the six are vertical and the second is when two fins are horizontal. Their results indicate that the fin orientation shows no significant effect on average Nusselt number, and the average Nusselt number increases with increasing Rayleigh number and decreases with increasing fin height.

Farinas *et al.* (1997) investigated the effect of internal fins on flow pattern, temperature distribution and heat transfer between concentric horizontal cylinders for different fin orientations and fin tip geometry for Rayleigh numbers ranging from 10^3 to 10^6.

They employed the two fin orientations used by Chai and Patankar (1993). They found that the second orientation presents a higher heat transfer rate than that of the first orientation. The fin tip geometry shows insignificant effect on heat transfer. Farinas *et al.* (1999) investigated laminar natural convection in horizontal bare and finned rhombic annulus for different fin numbers and lengths for Rayleigh numbers ranging from 10^3 to 10^7. They concluded that the heat transfer is maximized for a narrow cavity with two longer fins.

Rahnama *et al.* (1999) studied the effect of fins attached to the inner and outer cylinders of annuli on flow and temperature fields in a horizontal annulus. They studied the effect of fin orientation and height attached to the cylinders for Rayleigh numbers less than 10^6. They employed the two fin orientation used by Chai and Patankar (1993) and Farinas *et al.* (1997). Contrary to Farinas *et al.* (1997), their results show no effect of fin orientation on heat transfer as reported in Chai and Patankar (1993). They argued that the results of local and mean Nusselt number prediction show that the fin height has an optimum value for which the heat transfer rate is a maximum. Extending fin height beyond that value reduces heat transfer rate significantly which is due to the blocking effect of long fins on flow recirculation. Rahnama *et al.* (2002) investigated numerically the effect of two horizontal radial fins attached on the inner cylinder on the heat transfer for Rayleigh numbers less than 10^4. They reported that insertion of two fins reduced heat transfer compared to bare annulus.

Alshahrani and Zeitoun (2006) investigated the natural convection heat transfer between two horizontal concentric cylinders with two fins, at various inclination angles, attached to inner cylinder numerically using finite element technique. Laminar conditions up to Rayleigh number Ra_i of 5×10^4 were investigated. They concluded that the effect of fin length on heat transfer is higher in conduction-dominated zone than in convection-dominated zone. They found that using a solid fin of length ratio $L_R = 75$ per cent increases the heat transfer by 100 per cent in the conduction-dominated zone and by 20 per cent in the convection-dominated zone compared to bare annulus. They also found that heat transfer increases by 25 per cent when the fin inclination angle increases from horizontal to vertical position.

On the other hand, a great attention has recently been paid to study the effect of using porous materials on the fluid flow and heat transfer characteristics of many engineering systems. Kiwan and Al-Nimr (2001) investigated the effect of using porous fins to enhance natural convection heat transfer from a horizontal surface. They found that using porous fin with certain porosity may give same performance as conventional fin and save 100ε of the fin material. Abu-Hijleh (2003) investigated numerically the effect of using permeable fins on the outer surface of a horizontal cylinder. He concluded that using permeable fins provides much higher heat transfer rates than solid fins. However, he assumed that the fin thickness is very small ($d \rightarrow 0$) and it has a very high-thermal conductivity (i.e. $k_s \rightarrow \infty$). These assumptions eliminate the need for simulating the region inside the permeable (porous) fins. In the current work, these assumptions are relaxed. Therefore, the relevant continuity, momentum and energy equations are solved in the porous media. More recently, Kiwan (2006) proposed a simple method using Darcy's model to solve the natural convection heat transfer from a single porous fin attached to a vertical surface. He found that the simple model equation can predict the heat transfer rates obtained based on complex models within ± 10 per cent accuracy.

The experimental work in this area is very limited. Kim *et al.* (2000) investigated experimentally, the impact of using porous fins on the heat transfer and flow characteristics in plate fin heat exchanger. However, their experiment was conducted under forced convection heat transfer conditions.

The main objective of the present work is to study the effect of attaching porous fins at the inner cylinder on the heat transfer in the annulus and to compare the results with those obtained for solid fins. The effects of changing fin angles, Darcy number, Rayleigh number, and the thermal conductivity ratio on the heat transfer are investigated.

Problem formulation

The physical model of the investigated problem consists of two concentric cylinders trapping fluid in the resulting annular cavity as shown schematically in Figure 1. The inner cylinder is heated to a temperature higher than the outer cylinder temperature. Two porous fins of height of L_f and thickness of d are placed on the inner cylinder. The inner and outer annulus diameters are 20 and 80 mm, respectively. The fin length, L_f and thickness, d are kept constant in the current analysis, $L_f = 15$ mm and $d = 4$ mm. The angle between the two fins is 180°. The two fins are inclined an angle of θ with the horizontal axis, x_h. However, to reduce the effort in building the geometric models of current problem, the current problem is solved in the coordinate system x and y shown in Figure 1, where x is the coordinate placed along the fin and y is the coordinate placed normal to x-axis as shown in the figure.

The problem under consideration is steady, 2D and laminar. The porous medium is isotropic. All physical properties are considered to be constant except the density in the body force. The Boussinesq approximation is used to evaluate

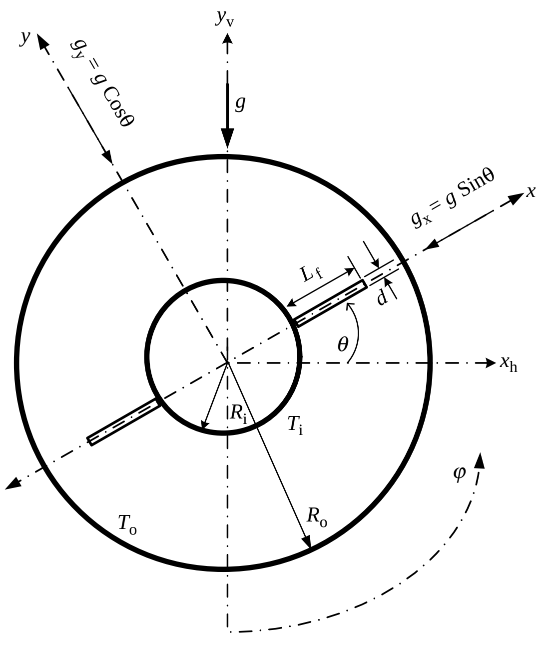


Figure 1.
Physical description and coordinate systems of the annulus with fins

the density in the body force term. The governing equations that describe the case are divided into two zones, the clear fluid and the porous zones. Therefore, two sets of equations are considered (continuity, momentum and energy equations). A set for clear domain (1) and another set for porous domain (2).

Governing equations

Mass conservation in clear domain (fluid) (1):

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0. \tag{1}$$

The momentum equations in the clear domain is:

$$\rho_f\left(u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y}\right) = -\frac{\partial p_1}{\partial x} + \mu\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) + \rho_f g \sin\theta, \tag{2}$$

$$\rho_f\left(u_1\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_1}{\partial y}\right) = -\frac{\partial p_1}{\partial y} + \mu\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}\right) + \rho_f g \cos\theta. \tag{3}$$

The energy equation in the clear domain is:

$$\rho_f c_p\left(u_1\frac{\partial T_1}{\partial x} + v_1\frac{\partial T_1}{\partial y}\right) = k_f\left(\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}\right). \tag{4}$$

Mass conservation in the porous domain (2) is:

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0. \tag{5}$$

Momentum equations in the porous domain (Darcy-Brinkman-Forchheimer equations):

$$\begin{aligned}\rho_f\left(u_2\frac{\partial u_2}{\partial x} + v_2\frac{\partial u_2}{\partial y}\right) = &-\frac{\partial p_2}{\partial x} + \mu\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2}\right) + \rho_f g \sin\theta - \frac{\mu}{K}u_2 \\ &-\frac{C_F\rho_f}{\sqrt{K}}\sqrt{u_2^2+v_2^2}u_2,\end{aligned} \tag{6}$$

$$\begin{aligned}\rho_f\left(u_2\frac{\partial v_2}{\partial x} + v_2\frac{\partial v_2}{\partial y}\right) = &-\frac{\partial p_2}{\partial y} + \mu\left(\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2}\right) + \rho_f g \cos\theta - \frac{\mu}{K}v_2 \\ &-\frac{C_F\rho_f}{\sqrt{K}}\sqrt{u_2^2+v_2^2}v_2.\end{aligned} \tag{7}$$

The energy equation in the porous domain:

$$\rho_f c_p\left(u_2\frac{\partial T_2}{\partial x} + v_2\frac{\partial T_2}{\partial y}\right) = k_{eff}\left(\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2}\right), \tag{8}$$

where, k_{eff} is the effective thermal conductivity. It could be estimated using $k_{eff} = k_f\varepsilon + (1-\varepsilon)k_s$. Other formulas can be used to estimate k_{eff} (Pavel and Mohamad, 2004). C_F is the Forchheimer coefficient, which can be taken as $C_F = 1.75/\sqrt{150\varepsilon^5}$ for beds of packed spherical particles.

Introducing the following dimensionless variables into the above equations:

$$U = \frac{uD_\mathrm{i}}{\alpha_\mathrm{f}}, \quad V = \frac{vD_\mathrm{i}}{\alpha_\mathrm{f}}, \quad X = \frac{x}{D_\mathrm{i}}, \quad Y = \frac{y}{D_\mathrm{i}}, \quad T^* = \frac{(T - T_\mathrm{o})}{(T_\mathrm{i} - T_\mathrm{o})}, \quad P = \frac{pD_\mathrm{i}^2}{\rho_\mathrm{f}\alpha_\mathrm{f}^2}, \quad (9)$$

the following sets of dimensionless equations are obtained.

For clear domain:

$$\frac{\partial U_1}{\partial X} + \frac{\partial V_1}{\partial Y} = 0, \quad (10)$$

$$U_1\frac{\partial U_1}{\partial X} + V_1\frac{\partial U_1}{\partial Y} = -\frac{\partial P_1}{\partial X} + Pr\left(\frac{\partial^2 U_1}{\partial X^2} + \frac{\partial^2 U_1}{\partial Y^2}\right) + Ra\,Pr\sin\theta T_1^*, \quad (11)$$

$$U_1\frac{\partial V_1}{\partial X} + V_1\frac{\partial V_1}{\partial Y} = -\frac{\partial P_1}{\partial Y} + Pr\left(\frac{\partial^2 V_1}{\partial X^2} + \frac{\partial^2 V_1}{\partial Y^2}\right) + Ra\,Pr\cos\theta T_1^*, \quad (12)$$

$$U_1\frac{\partial T_1^*}{\partial X} + V_1\frac{\partial T_1^*}{\partial Y} = \left(\frac{\partial^2 T_1^*}{\partial X^2} + \frac{\partial^2 T_1^*}{\partial Y^2}\right). \quad (13)$$

For porous domain:

$$\frac{\partial U_2}{\partial X} + \frac{\partial V_2}{\partial Y} = 0, \quad (14)$$

$$\left(U_2\frac{\partial U_2}{\partial X} + V_2\frac{\partial U_2}{\partial Y}\right) = -\frac{\partial P_2}{\partial Y} + Pr\left(\frac{\partial^2 U_2}{\partial X^2} + \frac{\partial^2 U_2}{\partial Y^2}\right) + Ra\,Pr\sin\theta T_2^* - \frac{Pr}{Da}U_2 - \Lambda\sqrt{U_2^2 + V_2^2}U_2, \quad (15)$$

$$\left(U_2\frac{\partial V_2}{\partial X} + V_2\frac{\partial V_2}{\partial Y}\right) = -\frac{\partial P_2}{\partial Y} + Pr\left(\frac{\partial^2 V_2}{\partial X^2} + \frac{\partial^2 V_2}{\partial Y^2}\right) + Ra\,Pr\cos\theta T_2^* - \frac{Pr}{Da}V_2 - \Lambda\sqrt{U_2^2 + V_2^2}V_2, \quad (16)$$

$$U_2\frac{\partial T_2^*}{\partial X} + V_2\frac{\partial T_2^*}{\partial Y} = k_\mathrm{r}\left(\frac{\partial^2 T_2^*}{\partial X^2} + \frac{\partial^2 T_2^*}{\partial Y^2}\right). \quad (17)$$

Boundary and interface conditions

The set of equations (10)–(17) are solved subjected to the following boundary conditions written in a non-dimensional form: no slip conditions at the walls, i.e. $U_1 = U_2 = 0$ at the inner and outer cylinders. The inner and outer surfaces of the annulus are assumed isothermals and are corresponding to $T_\mathrm{i}^* = 1$ and $T_\mathrm{o}^* = 0$, respectively.

The following conditions are applied at the interface between the porous and clear regions:

$$U_1 = U_2, \quad V_1 = V_2, \quad T_1^* = T_2^*, \quad \frac{\partial T_1^*}{\partial Y} = k_r \frac{\partial T_2^*}{\partial Y}, \tag{18}$$

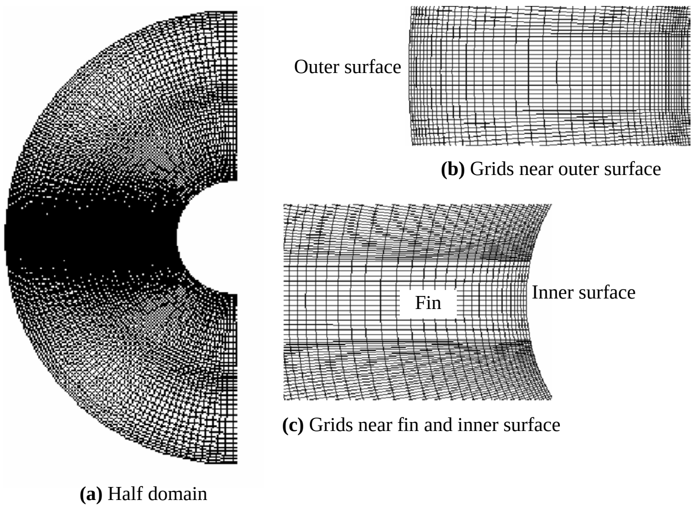
$$\left(\frac{\partial V_1}{\partial X} + \frac{\partial U_1}{\partial Y}\right) = \left(\frac{\partial V_2}{\partial X} + \frac{\partial U_2}{\partial Y}\right) \text{ and } \frac{\partial V_1}{\partial Y} = \frac{\mu_{eff}}{\mu}\frac{\partial V_2}{\partial Y}. \tag{19}$$

In this study, the effect of inertia losses is considered to be small and therefore it is neglected, therefore the value of Λ is taken as zero. Also, the value of Prandtl number is limited to air $Pr = 0.71$.

Numerical solution

The present problem is solved using a finite volume solver. A simple 2D mesh clustered near the inner, outer cylinders, and at the interface between clear fluid and porous medium was used. The pressure field is calculated using the SIMPLE algorithm as demonstrated in Versteeg and Malalasekera (1995). The hybrid-differencing scheme is used to difference the convective terms. The iterative solution is considered to have converged when the maximum of the normalized absolute residual across all nodes is less than 10^{-6}.

To verify the numerical code used in the current investigation, the results of the present code for clear annulus are tested and compared with the experimental results obtained by Grigull and Hauf (1966) and Kuehn and Goldstein (1976) and the numerical results of Alshahrani and Zeitoun (2005) as shown in Figure 2. The grid used in this validation is 80×360 as shown in Table I. It should be noted that k_e/k_f is the equivalent thermal conductivity ratio for clear annulus, where k_e represents the thermal conductivity that the stationary fluid should have to transfer the same amount of heat as the moving fluid. Ra_m is a modified Rayleigh number introduced by Alshahrani and Zeitoun (2005):



Note: Reproduced from the only available original

Figure 2.
Grid system

$$Ra_m = Ra_i^{1/4}\left(0.1389\left(1-\frac{D_i}{D_o}\right)+0.0927\right)\ln\left(\frac{D_o}{D_i}\right). \quad (20)$$

As shown in the figure, the predictions of the present code are in good agreement with experimental results of Grigull and Hauf (1966) and Kuehn and Goldstein (1976) and numerical data of Alshahrani and Zeitoun (2005). Furthermore, a comparison between the values of the average Nusselt numbers obtained by the present code and the numerical results obtained by Sathe *et al.* (1988) for the natural convection in an enclosure partially filled with a porous media is displayed in Table I. For this validation case, the enclosure aspect ratio is 5, the thickness of the porous material is (s_p) and the height of the enclosure is (s). In all test cases, a good agreement with the present code results is observed.

A four node quadrilateral element type was used in solving the current problem. The grid points are not distributed uniformly over the computational domain as shown in Figure 3. They have greater density near surfaces of the inner and outer surfaces of the annulus and the surface of the fins. The spacing expansion ratio, ER, of grid distribution along the radial and angular directions was selected 1.064 at the starting

Mesh	Solid fin (Q, (W/m))	Percentage change compared with 80 × 480 mesh	No fin (Q, (W/m))	Percentage change compared with 80 × 480 mesh	Porous fins $Da = 2.5 \times 10^{-4}$ (Q, (W/m))	Percentage change compared with 80 × 480 mesh
40 × 360	53.9938	2.8678	44.924	2.0196	76.78757	1.5081
60 × 360	53.177	1.3116	44.2394	0.46493	76.09737	0.5957
80 × 360	52.7437	0.4859	43.989	−0.10365	75.64678	0.0000
80 × 240	52.4888	0.0003	44.0344	0.0005	75.81564	0.2233
80 × 480	52.4886	0	44.0347	0	75.64676	0

Table I. Effect of mesh size on solution for $Ra = 5.3 \times 10^4$, $\theta = 0°$

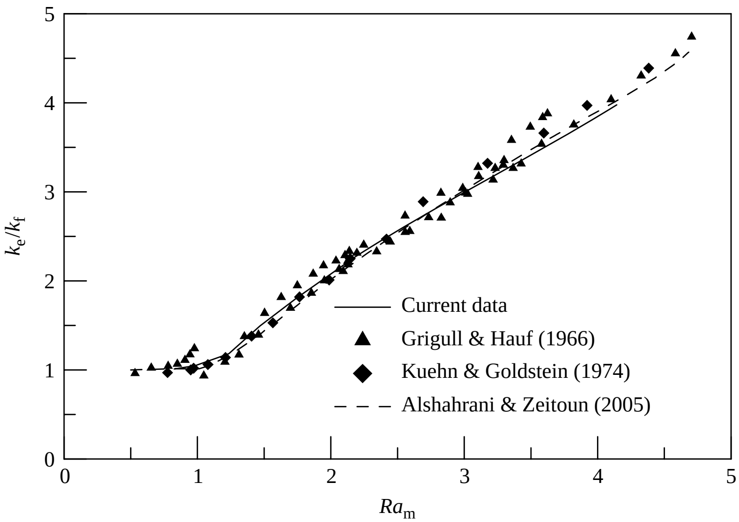


Figure 3. Comparison between current data of clear annulus and previous data

point of the boundary line and 0.94 at the end of the line. The expansion ratio is defined as the ratio of any two succeeding interval lengths. That is, $ER = l_{i+1}/l_i$ where l_i and l_{i+1} are the lengths of intervals i and $i + 1$, respectively.

In order to achieve a gird independent solution, extensive runs have been carried out for a range of mesh parameters. The change in the total heat transfer from the inner to the outer cylinder was found to be grid independent to better than one percent for a mesh system of 80 grids in the radial direction $\times$ 360 grids in the angular direction when $Ra = 5.3 \times 10^4$ and $\theta = 0°$ (for fin cases only). Table II shows the results when different grids are used to obtain the solution for solid fin, no fin and porous fin cases. Upon the results of this investigation a grid system of 80 $\times$ 360 is chosen to be used in the analysis.

However, it should be noted that the abovementioned meshing strategy failed to obtain a grid independent solution for some cases involving porous fins when $\theta \geq 67°$. The reason for that is mainly due to the nature of the flow. That is, varying the conductivity ratio or the angle of the porous fin changes the location of the temperature gradient around the outer cylinder. To overcome this problem an adaptive grid strategy is used. The main idea behind using adaptive refinement is to add grid points (cells) where they are needed in the mesh. The gradient-adaptation technique of Warren *et al.* (1991) used here allows adding grid points between the cells where the gradient of the temperature is higher than a value. To do this, the solution is started based on the grid size described earlier for few hundred iterations and then the gradient of the temperature at all cells is computed. Then, the maximum gradient is divided by 10. Cells (grid points) are then added between the cells where their temperature gradient is higher than the new value. Then, 30-50 more iterations are carried out and the above procedure is repeated until the total heat transfer for from the inner to the outer cylinder does not change within 1 per cent.

Results and discussion

The natural convection heat transfer in the enclosure formed between two concentric cylinders using porous fin at the inner cylinder is numerically investigated. The effect of using porous fins on the heat transfer was studied. Since, the intension of this study is to investigate the effect of using porous fins instead of using solid fins; all geometric parameters are fixed except the fin angle. These parameters include; the fin length, fin thickness, inner cylinder diameter, and outer cylinder diameter. The effect of

s_p/s	Results of Sathe *et al.* (1988)	Present results
0	3.725	3.84
0.1	3.157	3.14
0.2	2.717	2.68
0.3	2.539	2.57
0.4	2.483	2.51
0.5	2.468	2.46
0.6	2.467	2.46
0.7	2.465	2.46
0.8	2.387	2.43
0.9	2.094	2.12
1	1.825	1.85

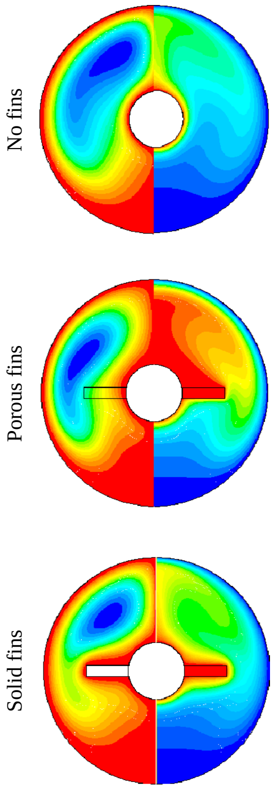
Notes: $Ra = 10^5$; $k_r = 1$; $Pr = 0.7$; $Da = 10^{-3}$; aspect ratio = 5

Table II. Comparison of the present code results with the results of Sathe *et al.* (1988)

changing the effective thermal conductivity of porous fins, fin angle, Darcy number and Rayleigh number are considered.

Figure 4 shows streamlines and isotherms inside the annulus. As shown in the figure, the existence of solid fin resists natural circulation in the annulus compared to finless annulus. However, the resistance of porous fins to the circulation is less than that of the solid fins. Consequently, the heat transfer from annulus equipped with porous fin will be enhanced. The isotherms in the bare annulus region indicate that the heat transfer is concentrated around the bottom of the inner cylinder and at upper part of outer cylinder. The existence of solid fins increases the thermal boundary layer thickness along the bottom of the inner cylinder and the attached fins. For the outer cylinder, the heat transfer intensifies along the upper parts. The existence of fins decreases the boundary layer along the outer cylinder surface. These results indicate that the heat transfer will increase as fin length increases. The flow through the porous fins, as shown in the figure, has the following effects on the nature of the flow in the gap between the two cylinders:

- it destroys the boundary layer along the upper part of the fins;
- it decreases boundary layer thickness along lower surfaces of the fins; and
- squeezes boundary layer along outer cylinder.

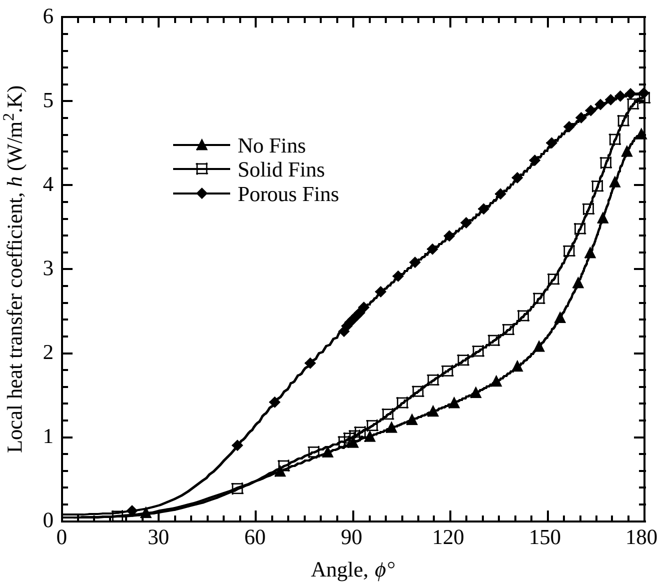


Notes: $Da = 2.5\times10^{-3}$; $k_r = 4\times10^{3}$; $Ra = 2.78\times10^{4}$

Figure 4.
Streamlines (left) and isotherms (right) for different configurations

Consequently, the presence of porous fins is expected to increase the heat transfer through the annulus. On conclusion, the above-mentioned clarification is shown in Figure 5. This figure indicates that comparing to no fins case, the use of solid (impermeable) fins enhances the heat transfer only in the upper-half of the outer cylinder. That is no enhancement in heat transfer can be noticed in the range $0 < \phi < 90$. Whereas, using porous fin enhances the flow in this region and the upper-half region. It should be noted that for $0 < \phi < 30$ almost no heat transfers to the outer cylinder.

The effect of Rayleigh number on heat transfer represented by Nusselt number is shown in Figure 6. As shown, Nusselt number increases as Rayleigh number increases. The results show the three distinct regimes of heat transfer introduced by Grigull and Hauf (1966). The first is the conduction dominated regime where the Nusselt number is approximately flat, the second is a transition regime and the third is the convection dominated regime, where Nusselt number is strongly dependent on Rayleigh number. Similar to no fin and solid fin cases, increasing Rayleigh number increases heat transfer. As Darcy number increases, the heat transfer increases. However, only small difference was observed upon increasing Darcy number from 2.5×10^{-3} to 2.5×10^{-2}. As shown in the figure, the solid fins increase heat transfer about 17 per cent, compared to no fin case, in the high-Rayleigh number region. However, porous fins of $Da = 2.5 \times 10^{-2}$, can increase the heat transfer up to 75 per cent, compared to solid fin case and about 100 per cent compared to no fin case. The results in the figure reveal that for low-Rayleigh numbers, no difference in heat transfer between solid and porous fins was observed. However, when $Da = 2.5 \times 10^{-4}$, the porous fin enhances heat transfer for Rayleigh number above 2,500 compared to solid fin, while for $Da = 2.5 \times 10^{-3}$ to 2.5×10^{-2}, it enhances heat transfer for Rayleigh number above 900. Therefore, increasing Darcy number enhances the heat transfer up to certain limit where a further increase in



Notes: $Da = 2.5 \times 10^{-3}$; $k_r = 4 \times 10^3$; $Ra = 2.78 \times 10^4$

Figure 5.
Distribution of heat transfer coefficients at the outer cylinder

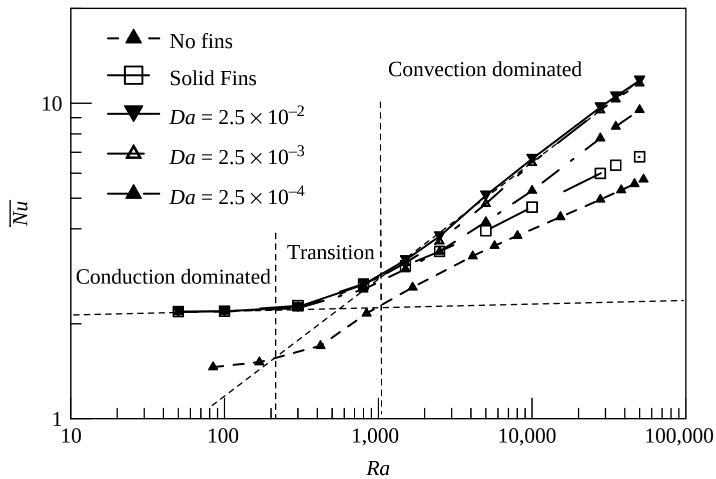
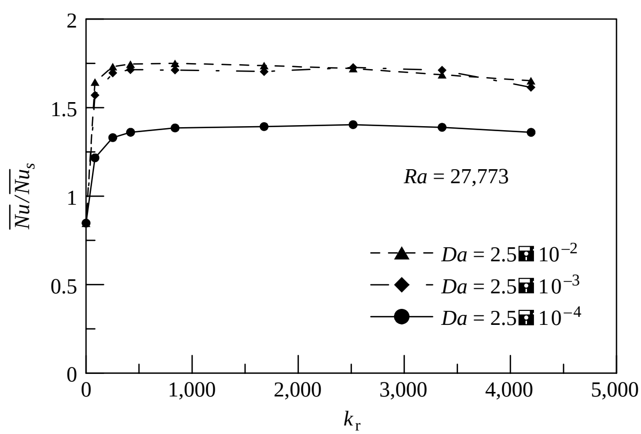


Figure 6. Variation of the average Nusselt number at the outer cylinder with the variation of *Ra* (for porous fins $k_r = 4 \times 10^3$)

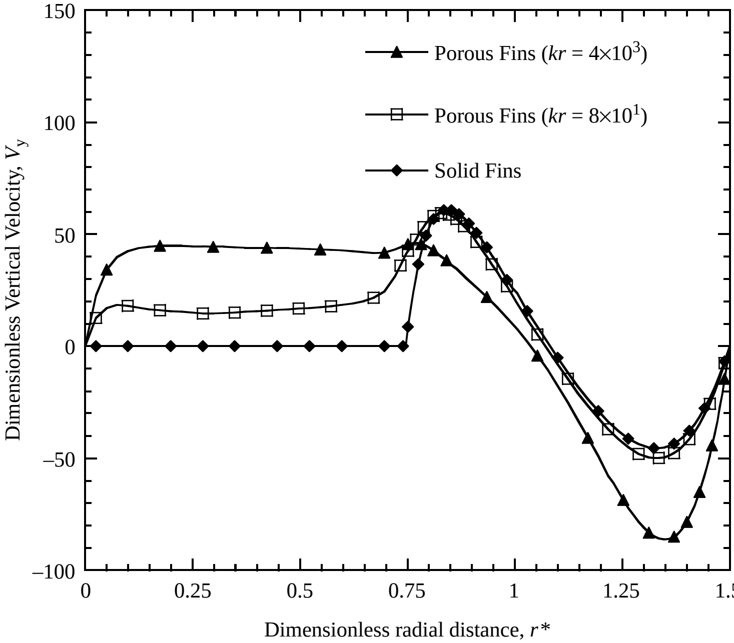
Darcy number, within the range investigated here, has no significant effect on Nusselt number.

Figure 7 shows the effect of varying the conductivity ratio, k_r, of the porous fin on Nusselt number compared with that of a solid fin for $k_s/k_f = 8,000$. It is obvious that as the conductivity ratio of the porous fin increases, the heat transfer ratio increases up to a certain limit beyond which a further increase in k_r has no significant effect on the heat transfer. For example, Figure 7 shows that increasing k_r beyond 850 has no significant impact on the heat transfer. Now, if we used the relation: $k_r = \varepsilon + (1 - \varepsilon)k_s/k_f$, to calculate the value of the porosity ε keeping in mind that $k_s/k_f = 8,000$, we find that $\varepsilon \approx 0.9$. This means that around 90 per cent of the material could be saved by using porous fin and still obtaining higher heat transfer rates. To better understand the effect of changing of k_r on the performance of the porous fin, refer to Figures 8 and 9.



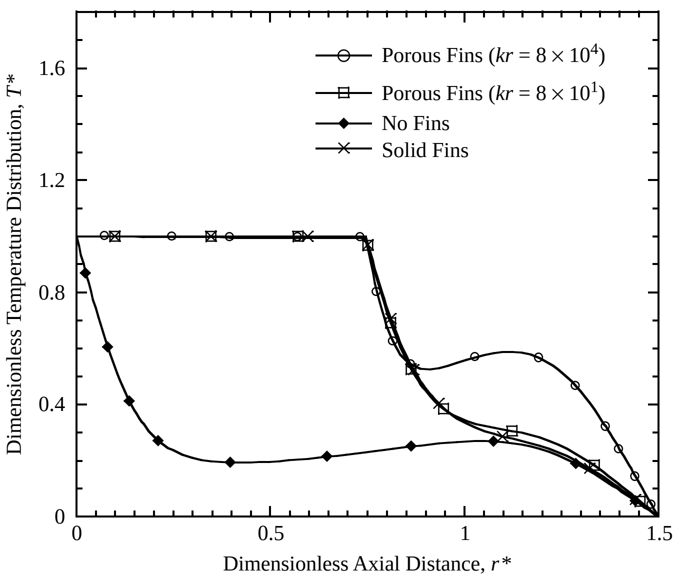
Note: *Ra* = 2.78×10^4

Figure 7. Effect of thermal conductivity ratio on Nusselt number ratio for different Darcy numbers



Notes: $\theta = 0°$; $Ra = 2.5\times10^4$

Figure 8.
The distribution of the dimensionless vertical velocity across the annulus

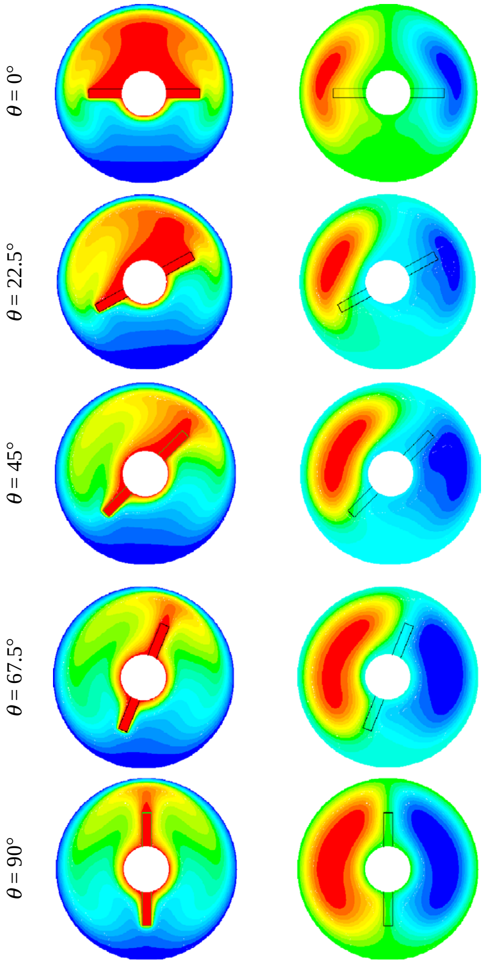


Notes: $\theta = 0°$; $Ra = 2.5\times10^4$

Figure 9.
The distribution of the dimensionless temperature across the annulus

These figures show the distribution of the dimensionless vertical velocity and dimensionless temperatures along the fin radial axis. It clearly shows that using solid fins reduces the convection current. Also, as the conductivity ratio of the porous fin increases, the temperature along the fin increases, as shown in Figure 9, and therefore, more fluid passes through the porous fin, as Figure 8 shows, this enhances heat transfer. This clearly illustrates that unlike solid fins, porous fins increases the heat transfer without paying the penalty of reducing convection current.

Figure 10 shows the isotherms (left) and streamlines (right) for different fin angles. As shown in the figure, unlike solid fin, the plume through the porous fins intensifies as the inclination angle decreases where it reaches the maximum at horizontal position of the fin. For the solid fin case, the heat transfer increases as the inclination angle increases where it reaches maximum at vertical position.



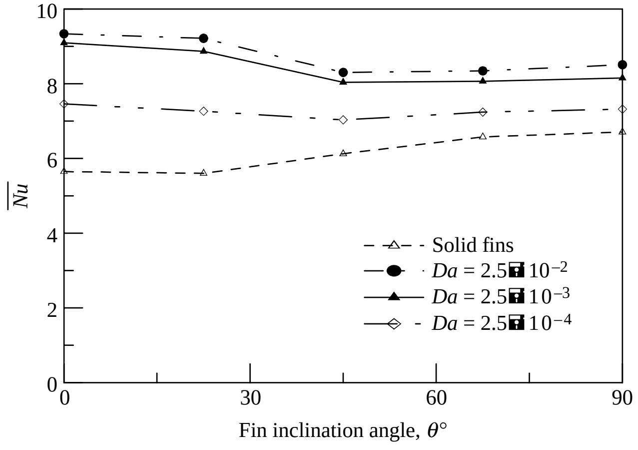
Notes: $k_r = 4\times10^3$; $Ra = 2.5\times10^4$

Figure 10. Isotherms (left) and streamlines (right) for different porous fin angles

Previous results of clear annulus (Grigull and Hauf, 1966; Kuehn and Goldstein, 1974, 1976; Alshahrani and Zeitoun, 2005) show that the fluid circulates in a kidney-shaped pattern and the circulation centers move up as Rayleigh number increases. The fluid moves up along the inner hot cylinder and falls down along the outer cold cylinder. The velocity is very low near the circulation centers. As the Rayleigh number increases, the boundary layer along the inner and outer cylindrical surfaces can be observed and high velocity and temperature gradients exist near the inner hot and outer cold surfaces. The shape of the streamlines indicates that the flow pattern intensifies as well as moves upward as the temperature difference increases. The results also shows that as the fin inclination angle increases the fins push the circulation up which squeezes the boundary layer along upper-half of the outer cylinder. This effect is similar to effect of increasing Rayleigh number in clear annulus, i.e. an increase in heat transfer. Results in Figure 11 show the effect of fin inclination angles on the heat transfer represented by average Nusselt number. As discussed before, the heat transfer increases as inclination angle increases for solid fins. However, for porous fins, the opposite is true, i.e. increasing inclination angle decreases heat transfer.

Conclusions

Natural convection heat transfer between two horizontal concentric cylinders with two porous fins attached to the inner cylinder was investigated numerically using a finite volume technique. Laminar conditions up to Rayleigh number of 5×10^4 were investigated. Effects of Darcy number, thermal conductivity ratio of porous fins and Rayleigh number on heat transfer through annuls for different fin inclination angles were investigated. It was found that increasing Rayleigh increases heat transfer through annulus, and increasing thermal conductivity ratio increases heat transfer till reaching maximum value then starts to slightly decrease. It is also found that increasing Darcy number increases the heat transfer up to certain limit where a further increase has no significant effect on the heat transfer within the investigated range of Darcy number. These results indicate that a high-heat transfer can be



Notes: $k_r = 4\times10^3$; $Ra = 2.5\times10^4$

Figure 11.
Variation of the average Nusselt number for different fin inclination angles

obtained while saving in fin material. Using porous fins increases heat transfer twice as its value of bare annulus and increases heat transfer 75 per cent of its value of annulus of solid fins. It is also found that unlike solid fins the rate of heat transfer from the cylinder equipped with porous fins decreases with increasing the fin angle.

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Corresponding author
O. Zeitoun can be contacted at: OZeitoun@ksu.edu.sa